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## COMBINATION EQUATIONS USED TO CALCULATE EVAPORATION AND POTENTIAL EVAPORATION

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## CONTENTS

	<i>Page</i>
Introduction .....	3
Development of equations .....	3
Energy budget equations .....	3
Transfer equations .....	4
Equations for evaporation .....	4
Bowen ratio method .....	4
Vapor pressure deficit combination formula .....	5
Evaluation of the transfer coefficient $h$ .....	6
A generalized combination method by Fuchs and Tanner .....	8
Evaporation equations using transport resistance concepts .....	8
Equations for potential evaporation .....	9
The combined concept .....	9
Potential evaporation using surface temperature .....	10
Summary .....	10
Literature cited .....	11
Appendix .....	12
Table of terms and units .....	12
List of major equations .....	13

# COMBINATION EQUATIONS USED TO CALCULATE EVAPORATION AND POTENTIAL EVAPORATION<sup>1</sup>

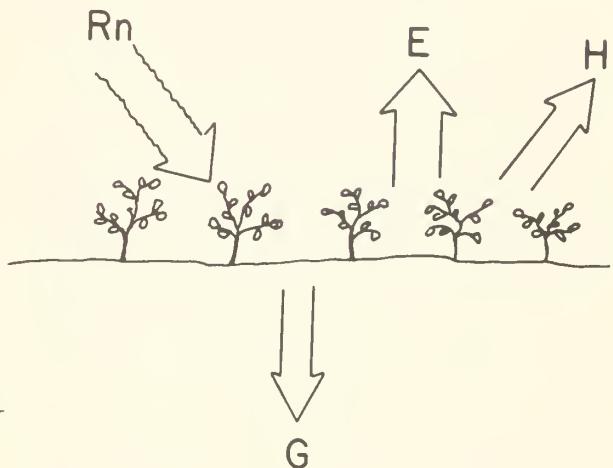
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## INTRODUCTION

Some of the best known and most recent evaporation equations were derived from basic energy balance and turbulent transfer principles using consistent nomenclature. Since few of the equations differentiate between soil evaporation and plant evaporation, the flux of water vapor away from any surface will simply be referred to as evaporation.

In figure 1 all arrows (fluxes of energy) toward the canopy are positive, while energy fluxes away from the canopy are negative; that is,  $R_n$  is positive while  $E$ ,  $H$ , and  $G$  are negative values in this figure (all symbols are defined in the appendix).

Figure 1.—Schematic diagram of fluxes of energy to and from the canopy.



## DEVELOPMENT OF EQUATIONS

### ENERGY BUDGET EQUATIONS

The vertical energy balance for a cropped surface can be given by

$$R_n + E + G + H = 0. \quad [1]$$

This equation is depicted in figure 1 where net radiation ( $R_n$ ) is adding energy to the canopy and it is being repartitioned into soil heat flux ( $G$ ), evaporation ( $E$ ), and sensible heat ( $H$ ). With well-watered fields and

moderate climates, the ratio of evaporation to net radiation ( $E/R_n$ ) is nearly equal to one while with a dry soil surface or stressed plants  $E/R_n < 1$  is expected with the ratio frequently as low as 0.5. If sensible heat is advected into a plot, the ratio can be greater than one and reported values as high as 1.5 are not uncommon.

The following assumptions are made in equation 1:

(1) The surface is large and uniform, that is, there is no horizontal divergence.

(2) Energy of photosynthesis is negligible. It is usually only 1 to 2 percent of net radiation, although Lemon has stated it can go as high as 5 percent during the most active growth stage of corn (7).<sup>3</sup>

(3) Energy stored in the crop volume can be neglected. This is generally valid (21), except possibly during a short period near sunrise and sunset.

<sup>1</sup>Contribution from the Soil and Water Conservation Research Division, Agricultural Research Service, U.S. Department of Agriculture, in cooperation with the Texas Agricultural Experiment Station.

<sup>2</sup>Research soil scientists, Soil and Water Conservation Research Division, Agricultural Research Service, at Weslaco, Tex.

<sup>3</sup>Underscored numbers in parentheses refer to Literature Cited, p. 11.

In assumption number 1, it is difficult to put numbers on what is a sufficiently large uniform surface to provide negligible horizontal divergences. In 1958, Elliott (4) analyzed height and fetch requirements, and reported that the maximum height ( $z_i$ ) for suitable measurement with a 95 percent profile adjustment could be found approximately by the equation  $z_i = 0.75X^{0.8}$  for  $X$  as a fetch distance in centimeters. This equation worked if the fetch was greater than 10 meters (m.). Rider (14) evaluated temperature, moisture, and profile variations downwind from a tarmac surface for the first 16 m. over a grass surface. The measurements showed that the greatest variation occurs in the first 10 to 15 m. after a discontinuity is encountered.

Dyer and Crawford, however, reported data for a well-watered site surrounded by dry fields in which they were able to measure horizontal variations as far as 230 m. downwind over the well-watered field (3). They also showed how evaporation and sensible heat transfer would vary from the leading edge inwards distances of 150 to 200 m. Generally, most of the measurable horizontal variations near the surface have dissipated in the first 100 m. Instruments above the crop need roughly 200 m. of fetch for each meter above the crop surface for a 90 percent profile adjustment.

$R_n$  is generally measured directly, but it also can be obtained from

$$R_n = Q_s - Q_{sr} + Q_a - Q_{ar} - Q_{bs} \quad [2]$$

if each of the components of  $R_n$  are known.

## TRANSFER EQUATIONS

The flux of heat through a solid ( $H_s$ ) can be determined by the equation

$$H_s = K \frac{\partial T}{\partial z}$$

where  $K$  is the coefficient of heat conduction (thermal conductivity) for the solid. This coefficient remains relatively constant over a wide temperature range for solids.

The turbulent vertical transfer of heat in air is given by

$$H = \rho C_p K_h \left[ \frac{\partial T}{\partial z} + \Gamma \right] \quad [3]$$

where  $\Gamma$  is the dry adiabatic lapse rate  $dT/dz = -0.01^\circ \text{C/m.}$  and can, therefore, be neglected in many applications of [3], since  $dT/dz$  over the earth's surface is generally 10 to 100 times greater than  $\Gamma$ , except at sunrise and sunset. The temperature in equation 3, strictly speaking, should have a bar over it, since it is a

time averaged value. However, since the temperatures and water vapor measurements referred to in this paper are all time averaged, no bars are used.

The equation for the turbulent flux density of latent heat is commonly seen in the following two forms for gradients of specific humidity and vapor pressure, respectively,

$$E = L_v \rho K_w \frac{\partial q}{\partial z} = L_v \rho K_w \frac{0.62}{P} \frac{\partial e}{\partial z} \quad [4]$$

where 0.62 is the ratio of the molecular weight of water to air. The flux of momentum or Reynolds' stress can be given as

$$\Upsilon = \rho K_m \frac{\partial u}{\partial z} = \rho u_*^2 \quad [5]$$

where the stress is assumed constant with height.

If the coefficients for turbulent heat, vapor, or momentum transfer in the air ( $K_h$ ,  $K_w$ , and  $K_m$ , respectively) were as constant as that for heat conductivity in a solid, the calculation of evaporation or sensible heat flux or both would be simple. However, the transfer coefficient above a surface depends on wind-speed, surface roughness, and air stability.

The roughness affects the size of the "turbulent parcels" and a hot surface affects the buoyancy and shape of the parcels. Because the transfer mechanism above the soil-plant interface is such a dynamic and complex process, the main problem concerning equations 3, 4, and 5 has involved the manner in which the transfer coefficients vary. In developing many of the following evaporation equations, the manner in which the transfer coefficients are handled is of key importance.

## EQUATIONS FOR EVAPORATION

### Bowen Ratio Method

Bowen (1) used a combination of the energy budget [1] and the transfer equations 3 and 4. In this method the ratio of  $H/E$  is used where

$$B = \frac{H}{E} = \frac{C_p P \ K_h \ \partial T / \partial z}{0.62 \ L_v \ K_w \ \partial e / \partial z}.$$

Assuming  $K_h = K_w$  and that the gradients are measured over the same interval, then upon integration

$$B = H/E = \gamma (\Delta T / \Delta e) \quad [6]$$

where  $\gamma = C_p P/0.62 L_v$ . Substituting [6] in [1] and rearranging gives

$$E = \frac{-(R_n + G)}{1 + \gamma (\Delta T/\Delta e)} \quad [7]^*$$

Asterisks are used beside the equation number of commonly used evaporation equations.

Equation 7 generally works well when the evaporative flux is large relative to sensible heat flux. Under these conditions  $\Delta T$  is small and  $\Delta e$  is large, so errors in the second term in the denominator have little effect on the calculated value for  $E$ . This evaporation equation has been used extensively by Frischen and van Bavel (5). Pruitt (13), Tanner (22), and others and is generally adequate for determining evaporation.

### Vapor Pressure Deficit Combination Formula

Slatyer and McIlroy (18) and Penman (12) derived an equation for evaporation that used [1], [3], and [4]. Equation 3 for this derivation is written in the integral form

$$H = \rho C_p h (T_a - T_o) \quad [8]$$

where  $h = K_h/z$  and [4] is written as

$$E = \frac{L_v \rho 0.62}{P} h (e_a - e_o). \quad [9]$$

Symbols with a subscript "o" refer to surface values.

The slope of the saturation vapor pressure curve for water ( $\Delta$ ) (19) can be written as  $\Delta = \partial e'/\partial T$  or relative to the terminology being used, the slope is closely approximated by

$$\Delta = \frac{(e'_a - e'_o)}{(T_a - T_o)} \quad [10]$$

for small differences.

Rearranging [10] and substituting it in [8] gives

$$H = \rho \frac{C_p}{\Delta} h (e'_a - e'_o) \quad [11]$$

where the primed values refer to saturated vapor pressure at the corresponding subscripted temperature.

Upon substituting the mathematical equalities  $e'_a = e_a + (e'_a - e_a)$  and  $e'_o = e_o + (e'_o - e_o)$ , we can write [11] as

$$H = \left[ \frac{C_p}{\Delta} \right] \rho h (e_a + e'_a - e_a - e_o - e'_o + e_o),$$

or

$$H = \left[ \frac{C_p}{\Delta} \right] \rho h (e_a - e_o) + \frac{C_p}{\Delta} \rho h \{ (e'_a - e_a) - (e'_o - e_o) \} \quad [12]$$

substituting  $(e_a - e_o)$  from [9] in the first term on the right side of [12] gives

$$H = \left[ \frac{\gamma}{\Delta} \right] E + \frac{C_p}{\Delta} \rho h \{ (e'_a - e_a) - (e'_o - e_o) \} \quad [13]$$

Substituting [13] into [1] yields

$$E = \frac{\Delta}{\Delta + \gamma} \left\{ - (R_n + G) - \rho \frac{C_p}{\Delta} h \{ (e'_a - e_a) - (e'_o - e_o) \} \right\} \quad [14]^*$$

This is a general form for the combination method involving vapor pressure deficits. In [14]  $(e'_a - e_a)$  is the water vapor saturation deficit at some height in the air while  $(e'_o - e_o)$  is the saturation deficit at the evaporating surface.

With different surface moisture conditions, [14] can be modified. If the surface is wet, then no saturation deficit exists and  $e'_o = e_o$ . Using this restraint in [14] yields the following equation for *potential evaporation*

$$E_p = \frac{\Delta}{\Delta + \gamma} \left[ - (R_n + G) - \rho \frac{C_p}{\Delta} h (e'_a - e_a) \right]. \quad [15]^*$$

In practice  $T_o$  is seldom known, so in using equations 14 and 15 an additional assumption is frequently made. It is assumed that  $T_a$  can be used for  $T_o$  and  $\Delta$  then is given by the slope of the saturated vapor pressure curve for water at  $T_a$ .  $T_a$  is also used in the calculation of  $\gamma$ . This assumption is believed to cause little error in the calculation of evaporation for well-watered surfaces (24).

A further modification of this equation is possible if evaporation is occurring from a wet surface with infinite fetch, since in this case the air near the surface is saturated and  $(e'_a - e_a) = 0$  and [15] reduces to

$$E_p = - \frac{\Delta}{\Delta + \gamma} (R_n + G). \quad [16]$$

Equation 16 can be put into the form of the Bowen ratio equation 7, where  $E$  becomes  $E_p$  from a wet surface with infinite fetch:

$$E_p = \frac{-(R_n + G)}{1 + \frac{\gamma}{\Delta}} = \frac{-(R_n + G)}{1 + \gamma} \frac{(T_a - T_o)}{(e'_a - e'_o)}. \quad [17]$$

### Evaluation of the Transfer Coefficient $h$

In evaluating the transfer coefficient  $h$  the relationship is really being determined between  $h$  and wind-speed, surface characteristics, and stability of the air. The development of this relationship generally starts by assuming an equality between the transfer of momentum and that of water vapor.

With this basic assumption in mind, a study of information available relating to the wind profile is helpful. The slope of the wind profile varies inversely with height above the ground. The relation is expressed by

$$\frac{\partial u}{\partial z} = \frac{u_*}{k} \frac{1}{z} \quad [18]$$

where the constant of proportionality is the frictional velocity ( $u_*$ ) divided by von Karman's constant ( $k \approx 0.4$ ). Integration of [18] from  $u = o$  at  $z_o$  to  $u_a$  at  $z-d$

$$\int_o^{u_a} \partial u = \frac{u_*}{k} \int_{z_o}^{z-d} 1/z \partial z$$

gives

$$u_a = \frac{u_*}{k} \ln \frac{z-d}{z_o} \quad [19]$$

where  $z_o$  is the roughness length and  $d$  is the zero plane displacement as shown in figure 2. Values for  $z_o$  and  $d$  can be obtained from wind profile data using a least squares fit of the data to a wind profile equation such as [19] or by graphically fitting the best curve to the data. The graphical fitting of the data is shown in figure 3. Windspeeds above a 140 centimeter (cm.) tall cotton canopy with neutral temperature conditions are shown in part A and the same data is shown in part B with a  $d$  value of 60 cm. The height is plotted on a logarithmic scale for both A and B. Extrapolation of the profile to  $u = o$  gives a  $z_o$  value of 19 cm.

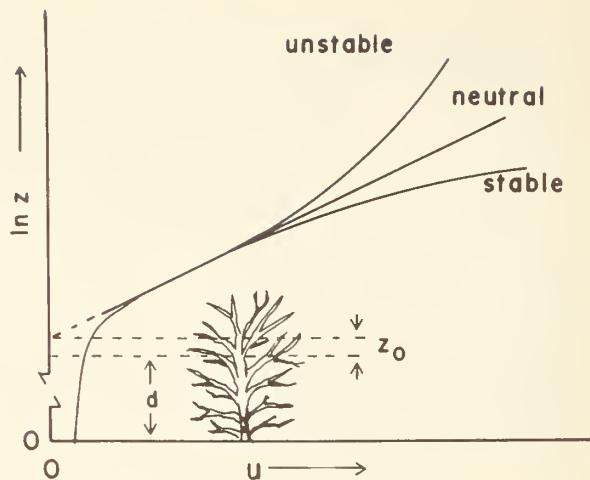


Figure 2.—Schematic representation of windspeed, distribution in and above a vegetative system with different stability regimes.

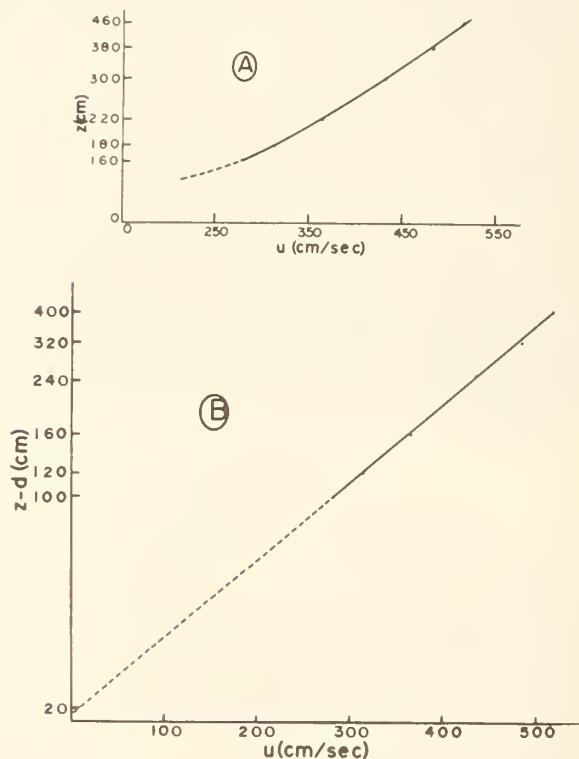


Figure 3.—Wind profile data above a 140-cm. high cotton canopy: A, under neutral conditions; and B, with a  $d$  value of 60 cm.

Unfortunately [19] holds only for neutral (adiabatic) conditions as shown in figure 2 and not for unstable (diabatic, lapse) conditions or during stable (subadiabatic, inversion) conditions. Thus, the log law holds only for neutral conditions and does not adequately incorporate other stabilities which it must include to be of general applicability (25).

Equation 18 can be generalized by including a stability function as follows:

$$\frac{\partial u}{\partial z} = \frac{u_*}{k} \frac{\phi}{z} . \quad [20]$$

Upon integration

$$u_a = \frac{u_*}{k} \int_{z_o}^{z-d} \phi/z \, dz . \quad [21]$$

One form given for  $\phi$  that is frequently referenced was suggested by Monin and Obukhov (9) where

$$\phi = 1 + \alpha \frac{z}{L} , \quad [22]$$

$$\text{and } L = -T \rho C_p u_*^3 / (kgH)$$

in which  $T$  is an absolute temperature and  $\alpha$  is an empirical constant. Another form for  $\phi$  involving Richardson's number ( $Ri$ ) for modification of the logarithmic wind equation with diabatic conditions is

$$\phi = (1 + b Ri)^a , \quad [23]$$

and

$$Ri = \frac{g}{T} \ln \frac{(z - d)}{z_o} (T_a - T_o) u_a^{-2} z . \quad [24]$$

Lettau (8) has tabulated the different values for  $a$  and  $b$  obtained in six research studies. Other forms of  $\phi$  are numerous and Sellers (16, p. 153) states, "At least eight different 'diabatic profiles' have appeared in the literature, each one supposedly fitting the observations."

The values for  $a$  and  $b$  that are most likely to be recommended are  $a = -0.25$  and  $b = -18$ . Thus [23] is

$$\phi = (1 - 18 Ri)^{-0.25} . \quad [25]$$

Equation 25 has been derived independently by five authors and has been termed the "KEYPS" profile.

Since the transfer coefficient  $h$  is in [9], which is [4] integrated, we will continue the evaluation of  $h$  by reintegrating [4]

$$E \int_{z_o}^{z-d} 1/K_w \, dz = j \int_{e_o}^{e_a} \, de$$

which upon integration with  $j = .62 L_v \rho / P$ , and rearranging

$$E = j (e_a - e_o) \left[ \int_{z_o}^{z-d} 1/K_w \, dz \right]^{-1} \quad [26]$$

but  $K_w$  as a function of  $z$  is not known.

However, replacing  $\frac{\partial u}{\partial z}$  in [5] with [20] gives

$$K_m = k u_* \frac{z}{\phi} . \quad [27]$$

Using the assumption stated above that  $K_m = K_w$  allows the substitution of [27] in [26]

$$E = j k u_* (e_a - e_o) \left[ \int_{z_o}^{z-d} \frac{\phi}{z} \, dz \right]^{-1} . \quad [28]$$

Since  $u_*$  may be difficult to evaluate, it can be substituted from [21] which gives

$$E = k^2 u_a j (e_a - e_o) \left[ \int_{z_o}^{z-d} \frac{\phi}{z} \, dz \right]^{-2} . \quad [29]$$

Setting [9] equal to [29] gives

$$h = \frac{k^2 u_a}{\left[ \int_{z_o}^{z-d} \frac{\phi}{z} \, dz \right]^2} . \quad [30]$$

A form of  $h$  found in the literature (16) can be obtained by simply letting

$$\frac{\phi}{z} = \frac{\phi - 1}{z} + \frac{1}{z}$$

in [30] which upon integration gives

$$h = \frac{k^2 u_a}{\left[ \Phi + \ln \frac{z - d}{z_o} \right]^2} , \quad [31]$$

$$\text{where } \Phi = \int_{z_o}^{z-d} \frac{\phi - 1}{z} \, dz . \quad [32]$$

The expression for  $h$  in [31] can be simplified for neutral stability. With neutral stability,  $\phi = 1$  and [31] reduces to the logarithmic law for the wind profile. Likewise, [20] reduces to [18], and [25] reduces to  $\phi = 1$  since  $Ri$  reduces to zero as shown by [24] because in neutral conditions  $T_a \simeq T_o$ . With  $\Phi = 0$ , [31] can be rewritten as

$$h = \frac{k^2 u_a}{\left[ \ln \frac{z-d}{z_o} \right]^2} = C_a u_a \quad [33]$$

where  $C_a$  is the drag coefficient.

Over water, well-watered vegetation, or wet soil, the temperature profile is frequently near adiabatic. Under these conditions and when working near the crop surface,  $h$  as evaluated from [33] is generally believed to work satisfactorily.

Under stable conditions, [22] has frequently been found to best represent the atmospheric stability correction factor; however, since stable conditions exist most frequently at night when evaporation is small or negligible, stable conditions can be considered minor.

An exception to this general statement may exist when oasis (advective) conditions exist and hot air blows over a cool, well-watered, rapidly evaporating crop.

For unstable conditions,  $h$  is evaluated from [31] which requires the integration of [32]. Using numerical integration, Lettau (8) developed the table of  $h$  values for different stability conditions, and Tanner (22) has also published the same table. Fuchs and Tanner (6) reported that  $h$  values calculated with [31] under unstable conditions above a drying soil averaged 25 percent larger than  $h$  values obtained from [33].

#### *A Generalized Combination Method by Fuchs and Tanner*

Fuchs and Tanner have proposed a combination formula relating  $E$  to  $E_p$  in which  $E_p$  and then  $E$  are evaluated (6). Their equation can be obtained by using [15] in [14] and using the identity  $(e'_o - e_o) = (e'_o - e_a) - (e_o - e_a)$  to yield

$$E = E_p + \frac{\rho C_p}{(\Delta + \gamma)} h \quad [ (e'_o - e_a) - (e_o - e_a) ] . \quad [34]$$

Then substituting from [9] for  $-h (e_o - e_a)$  gives

$$E = E_p + \frac{\rho C_p}{\Delta + \gamma} \left[ h (e'_o - e_a) + E / \frac{L_v \rho .62}{P} \right] . \quad [35]$$

Rearranging gives

$$E = \left[ \frac{\Delta + \gamma}{\Delta} \right] E_p + \frac{\rho C_p}{\Delta} h (e'_o - e_a) . \quad [36]^*$$

The authors used [31] to obtain  $h$  in [36].  $G$  and  $T_o$  are the only surface measurements required in this equation. The larger  $T_o$  is, the larger the second term on the right becomes and the smaller  $E$  becomes, remembering that evaporation always has a negative value when the vapor flux is away from the surface. Physically this is to be expected since as  $E$  decreases, with the same  $R_n$ , the energy must be partitioned differently. An increasing  $T_o$  allows this to occur since  $H$  and  $G$  away from the surface can increase as well as  $Q_{bs}$ . Fuchs and Tanner found a good comparison between the  $E$  values from [36] and those from the Bowen ratio method for a soil with a nearly dry surface.

For smooth soil or water surfaces, the surface temperature can generally be used as  $T_o$ . With crops or rough surfaces, however, it should be kept in mind that  $T_o$  is the air temperature at  $d + z_o$ , since this is the lower limit used in the integration of the equations used in deriving most of the evaporation equations. Thus, a true  $T_o$  or  $e_o$  can be obtained only by an extrapolation of air temperature or water vapor pressure profiles to the height  $(d + z_o)$ . Thus, equation 36 works best for nonvegetative surfaces.

#### *Evaporation Equations Using Transport Resistance Concepts*

Several investigators (10, 20) consider the resistance concept as shown in figure 4 of value.

Evaporation equations for the fluxes involved can be written as

$$E = j \left[ \frac{(e_o - e_a)}{r_a} \right]$$

$$E = j \left[ \frac{(e_i - e_o)}{r_i} \right]$$

$$E = j \left[ \frac{(e_i - e_a)}{(r_i + r_a)} \right]$$

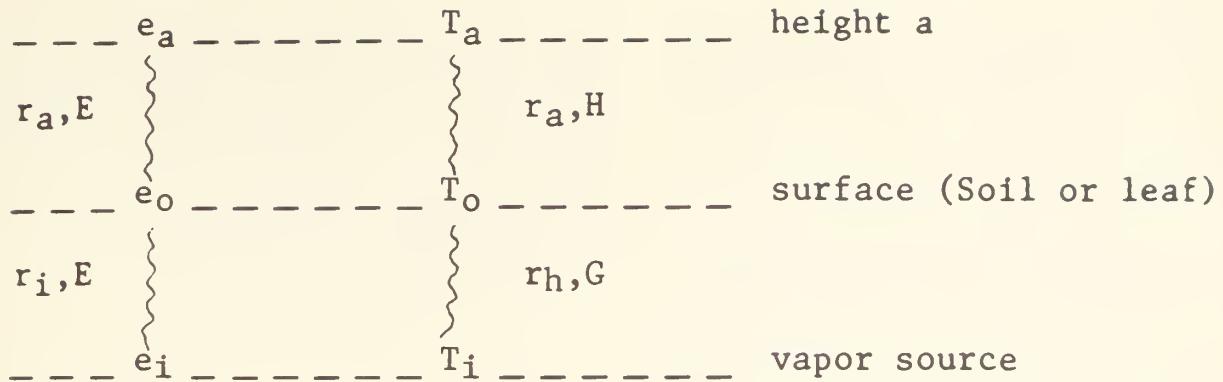


Figure 4.—Resistive network model of the transport of heat and vapor in the atmospheric surface layer and through the substrate surface layer.

Here  $r_a = 1/h$  for the air resistance. However, if a canopy is involved, an additional resistance for the canopy should be included. Also, a resistance value for a laminar flow surface layer and leaf stomata for cropped surfaces may need to be included in some cases. Although this approach may be appealing, there are possible inaccuracies in the model since evaporation from the soil as well as the canopy takes place from a vertically distributed array of sources as opposed to the one horizontal vapor source postulated in resistance models. Tanner and Fuchs (23) have discussed some of the problems with resistance models.

### EQUATIONS FOR POTENTIAL EVAPORATION

Evaporation equations provide a measure of water being lost with a given atmospheric demand. Potential evaporation equations, on the other hand, yield evaporation that would occur with a given atmospheric demand from a "wet surface"; that is a water surface or a field recently watered where there is no restriction on the availability of water for evaporation. In the equations discussed in this paper, if the development of the equation involves having  $e_o$  be the saturated vapor pressure at  $T_o$ , the equation assumes a wet surface and the equation predicts potential evaporation.

#### The Combined Concept

One potential evaporation equation, [15], was developed in the discussion of vapor pressure deficit formula. Van Bavel (24) used this equation and evaluated  $h$  with equation 33, which gives

$$E_p = \frac{\Delta}{\Delta + \gamma} \left\{ -(R_n + G) \right. \quad [37]$$

$$- \rho \frac{C_p}{\Delta} k^2 u_a \left[ \ln \frac{z - d}{z_o} \right]^{-2} (e'_a - e_a) \} \quad [37]^*$$

With this concept no measurements of surface temperature are required; however, this may cause some error in  $\Delta$ . Possible effects of this error in  $\Delta$  were evaluated and found to cause only a very small error in  $E_p$ .

The use of [37] by other authors has cast some doubt on the validity of the form of  $h$  used in this equation during windy conditions. In particular with a value of  $z_o$  of 1 cm. or greater, the second term becomes very important. Skidmore (17) reported that in Kansas for average daily windspeeds of 0.88 and 2.26 m/sec, measured at 45-cm. height, the wind-dominant term (second term) contributed 33 and 113%, respectively, as much as the radiation-dominant term to the total calculated potential evaporation. For these two consecutive days, the ratio of  $E_p$  from [37] to net radiation was 0.98 and 1.60, respectively, while the ratio of  $E_p$  over  $E$  from the Bowen ratio were 1.16 and 1.62.

Parmele (11) found that during midday equation 37 tended to overestimate evaporation from bare wet soil by 10 to 25 percent and evaporation was also generally overestimated for perennial ryegrass. However, these were truly overestimates only if the measured evaporation was occurring at a potential rate. Parmele found when using 24-hour average values in [37] that calculated and measured daily evaporation were very close. Using  $z_o$  values suggested by van Bavel on crops like well-watered alfalfa and daily values of the required parameters, Jensen<sup>4</sup> found total calculated evaporation for the season to be about two times net radiation, which was not realistic. Using a value of 0.5 to 1.0 cm. for  $z_o$ , however, yielded values that compared favorably with measured values with alfalfa. Good results from the equation have been reported for both hourly and daily values over open water, wet soil, and alfalfa when measurements taken at 2 m. were used (24).

<sup>4</sup> Personal communication from Marvin E. Jensen.

## Potential Evaporation Using Surface Temperature

Conaway and van Bavel (2) proposed the use of equation 9 with  $h$  determined from [33], which gives

$$E_p = jk^2 u_a \left[ \ln \frac{z - d}{z_o} \right]^{-2} (e_a - e_o). \quad [38]*$$

This method is based entirely on mass or vapor transfer equations and does not involve the energy balance

## SUMMARY

In deciding which evaporation equation will work best in a particular project, several criteria should be evaluated, including the following:

(1) Is evaporation or potential evaporation to be evaluated?

(2) Are instruments required to make the measurements available, and can all the variables required in the model be measured with sufficient accuracy? Can data reduction and calculations be handled without delay and considerable man-hours?

(3) Is the evaporation equation based on a physical or empirical model and has the equation been sufficiently evaluated? If not, are lysimeters or other means available to adequately check the equation? Only equations derived basically from physical models have been discussed in this publication; however, Rosenberg has recently reviewed the broad scope of evaporation equations (15).

(4) What assumptions and boundary conditions have been used in developing the equation, and can these conditions be met in this particular research project?

Certainly other criteria could be suggested, but these four should be sufficient for choosing the best evaporation equation to be used in a particular project.

Whether an evaporation or potential evaporation equation is needed will depend entirely upon the type of research project. Certainly a water surface as well as many well-watered crops and soils will evaporate at or near their potential rates. Evaporation will be less than potential when any restriction on the availability of water for evaporation exists. Frequently the  $E_p$  equations are less complex than actual evaporation equations.

All of the required measurements for the evaporation equations derived can be made; however, the ease with which the different measurements can be made varies considerably. Of the energy flux values required, net radiation ( $R_n$ ) is probably the easiest to measure, but good instrument calibration, maintenance, and periodic recalibration are required. Since  $R_n$  is large, small percentage errors in it greatly affect  $E$ . Soil heat flux  $G$  has a much smaller value than  $R_n$  ( $G < \frac{R_n}{10}$ , generally) so

equation. In this approach surface temperature is required, and Conaway and van Bavel suggest using infrared thermometer techniques for determining its value. They define  $e_o$  as surface vapor pressure; however, it probably would be more proper to define  $e_o$  in this potential evaporation equation as the saturated vapor pressure at  $T_o$ .

This equation gave accurate values of  $E_p$  from a wet soil surface both on an hourly and daily basis.

larger percentage errors in  $G$  can be tolerated.  $G$  can be obtained from soil heat flux plates, which have been calibrated in the soil with a moisture content near that expected during the experiment, or by calculation from soil temperature and moisture measurements and a knowledge of some physical properties of the soil.

Vapor pressure of the air ( $e_a$ ) is frequently obtained from wet- and dry-bulb measurements, LiCl probes, infrared adsorption, or dew cells, and any of these methods can be sufficiently accurate. All methods for determining water vapor, however, require extreme care and considerable maintenance. Saturated vapor pressure for a smooth wet surface  $e'_o$  is obtained with ease if surface temperature is known, since  $e'_o = f(T_o)$ . If  $E < E_p$  then  $e'_o \neq e_o$ , and  $e_o$  is very difficult to determine accurately.

Air temperature ( $T_a$ ) is relatively easy to measure but  $T_o$  may be more difficult because direct contact measurements of surface temperatures are seldom acceptable under natural conditions. Remote determination via infrared thermometry, however, can be used for smooth surfaces to determine  $T_o$  if emissivity of the surface and background radiation in the appropriate wavelength interval are known. Infrared determination of  $T_o$  makes possible the use of equation 36 to determine  $E$  and equation 38 to evaluate  $E_p$ .

In the Bowen ratio method, gradients in the air of  $\Delta e$  and  $\Delta T$  may be required. Over the intervals for which the gradients are determined—usually an interval of only 50 to possibly 200 cm.—the differences in temperature and vapor pressure may be quite small; thus, considerable accuracy is required. With a well-watered surface  $\Delta e$  may be large, while  $\Delta T$  will usually be small and should be known to at least  $\pm 0.02^\circ$  C. For a surface or crop that is quite dry,  $\Delta T$  will generally be large, while  $\Delta e$  will be small. Since  $\Delta e$  appears in the denominator of equation 7, when it is a small fraction it greatly affects the calculated value for  $E$ . Because of the accuracy required in obtaining these gradients, thermopiles or other very sensitive instruments for obtaining temperature differences are used, and the sensors at the two levels are periodically interchanged to eliminate instrument bias.

Evaporation equations that require an evaluation of the turbulent transfer coefficient  $h$  require a continuous measure of  $u_a$  and periodic wind profile measurements with at least six sensitive cup anemometers. Wind profile measurements under neutral conditions are required for the evaluation of  $d$  and  $z_o$ . Also, for some crops  $z_o$  appears to be a function of windspeed, so profiles with a variety of wind conditions may be required as well as periodic measurements as the surface changes in roughness or as the crop grows. Since a number of values have been reported for  $z_o$  and  $d$  in the literature for various crops, use of "book" values for  $z_o$  and  $d$  may be possible.

For rapid data reduction and analysis, a complete system analysis incorporating the entire system from

initial fabrication or purchase of sensors through experimental setup, data recording systems, equipment maintenance, computer computations, printout, and plotting is a must.

Various empirical evaporation equations have been used extensively and some have been very valuable in providing estimates of evaporation. Nearly without exception the empirical equations do best in locations similar in climate to where the equations were developed; in other climates, these equations may yield unrealistic estimates of evaporation. Evaporation equations based on physical models should be generally applicable to all areas, if the models are sound and the conditions used in developing the equation are met.

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## APPENDIX

TABLE OF TERMS AND UNITS

Symbol	Description
$C_a$	$= k^2 \left[ \ln \frac{(z - d)}{z_o} \right]^{-2}$ drag coefficient (dimensionless)
$C_p$	specific heat of air at constant pressure (cal. g. <sup>-1</sup> °C. <sup>-1</sup> )
$d$	height displacement parameter (cm.)
$E$	latent heat flux density (evaporation) (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )
$E_p$	potential evaporation flux density (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )
$e$	water vapor pressure (mb)
$e_a$	vapor pressure in the air at height $a$ (mb)
$e'_a$	saturated water vapor pressure at $T_a$ (mb)
$e_i$	water vapor pressure at $T_i$ (mb)
$e_o$	water vapor pressure at surface (mb)
$e'_o$	saturated water vapor pressure at $T_o$ (mb)
$G$	soil heat flux density (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )
$g$	acceleration of gravity (980 cm. sec. <sup>-2</sup> )
$H$	sensible heat flux density (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )

Symbol	Description
$h$	transport coefficient for heat and water vapor (cm. min. <sup>-1</sup> )
$j$	$= 0.62 L_v \rho / P$ (cal. mb <sup>-1</sup> cm. <sup>-3</sup> )
$K$	thermal conductivity (cal. min. <sup>-1</sup> cm. <sup>-1</sup> °C. <sup>-1</sup> )
$K_m$	transfer coefficient for momentum (cm)
$K_h$	transfer coefficient for heat (cm. <sup>2</sup> min. <sup>-1</sup> )
$K_w$	transfer coefficient for water vapor (cm. <sup>2</sup> min. <sup>-1</sup> )
$k$	von Karman constant ( $\approx 0.4$ ) (dimensionless)
$L_v$	latent heat of vaporization (cal./g.)
$P$	atmospheric pressure (mb)
$Q_a$	incoming thermal radiation flux density from the atmosphere (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )
$Q_{ar}$	reflected thermal radiation flux density (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )
$Q_{bs}$	emitted thermal radiation flux density (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )
$Q_s$	incoming solar radiation flux density (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )
$Q_{sr}$	reflected solar radiation flux density (cal. cm. <sup>-2</sup> min. <sup>-1</sup> )

Symbol	Description	Symbol	Description
$q$	specific humidity (dimensionless)	$\Delta T$	temperature difference between two levels in the air ( $^{\circ}\text{C}$ )
$R_n$	net radiation flux density (cal. $\text{cm.}^{-2}$ $\text{min.}^{-1}$ )	$\Upsilon$	flux density of momentum (g. $\text{cm.}^{-2}$ $\text{min.}^{-1}$ )
$r_a$	transport resistance between the surface and height $a$ ( $r_a = 1/h$ ) (sec. $\text{cm.}^{-1}$ )	<b>LIST OF MAJOR EQUATIONS</b>	
$r_h$	internal resistance of the surface layer to the transport of heat (sec. $\text{cm.}^{-1}$ )	Energy balance equation	
$r_i$	internal resistance of the surface layer to the transport of water vapor (sec. $\text{cm.}^{-1}$ )	$R_n + E + G + H = 0 \quad [1]$	
$T$	temperature ( $^{\circ}\text{C}$ )	Turbulent heat transfer equation	
$T_a$	air temperature at height $a$ ( $^{\circ}\text{C}$ )	$H = \rho C_p K_h \left[ \frac{\partial T}{\partial z} + \Gamma \right] \quad [3]$	
$T_o$	surface temperature ( $^{\circ}\text{C}$ )	or	
$u$	windspeed (cm./min.)	$H = \rho C_p h (T_a - T_o) \quad [8]$	
$u_a$	windspeed at height $a$ (cm./min.)	Latent heat transfer equation	
$u_*$	frictional velocity (cm./min.)	$E = L_v \rho K_w \frac{\partial q}{\partial z} = L_v \rho K_w \frac{.62}{P} \frac{\partial e}{\partial z} \quad [4]$	
$z$	height in the air from the soil surface (cm.)	or	
$z_i$	height above the crop to which the profile has been modified to 95 percent of its final value (cm.)	$E = \frac{L_v \rho .62}{P} h (e_a - e_o) \quad [9]$	
$z_o$	roughness height (cm.)	Bowen ratio method	
$\alpha$	constant in Monin and Obukhov wind profile equation (dimensionless)	$E = \frac{-(R_n + G)}{1 + \gamma \frac{\Delta T}{\Delta e}} \quad [7]^*$	
$\gamma$	$= \frac{C_p P}{L_v .62}$ (mb $^{\circ}\text{C.}^{-1}$ ) psychometric constant	Vapor pressure deficit combination formula	
$\Gamma$	dry adiabatic lapse rate ( $^{\circ}\text{C. cm.}^{-1}$ )	$E = \frac{\Delta}{\Delta + \gamma} \left\{ -(R_n + G) - \rho \frac{C_p}{\Delta} h \{ (e'_a - e_a) - (e'_o - e_o) \} \right\} \quad [14]^*$	
$\Phi$	$= \int_{z_o}^{z-d} \frac{\phi-1}{z} \partial z$ (dimensionless)	A generalized combination method by Fuchs and Tanner	
$\phi$	stability profile influence function (dimensionless)	$E = \frac{\Delta + \gamma}{\Delta} E_p + \frac{\rho C_p}{\Delta} h (e'_o - e_a) \quad [36]^*$	
$\rho$	density of air (g. $\text{cm.}^{-3}$ )	where	
$\Delta$	slope of the saturation water vapor curve versus temperature (mb $^{\circ}\text{C.}^{-1}$ )	$h = k^2 u_a \left[ \Phi + \ln \frac{z-d}{z_o} \right]^{-2} \quad [31]$	
$\Delta e$	vapor pressure difference between two levels in the air (mb)		

and  $E_p$  is calculated from the vapor pressure deficit equation 15.

The combined concept (for  $E_p$ )

where

$$h = k^2 u_a \left[ \ln \frac{(z - d)}{z_o} \right]^{-2} = C_a u_a \quad [33]$$

Potential evaporation using surface temperature

$$E_p = \frac{\Delta}{\Delta + \gamma} \left[ -(R_n + G) - \frac{\rho C_p}{\Delta} h (e'_a - e_a) \right] \quad [15]^*$$

$$E_p = jk^2 u_a \left[ \ln \frac{z - d}{z_o} \right]^{-2} (e_a - e_o) \quad [38]^*$$





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